

# **Decision Tree and Related Analytics**

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#### ROV Technical Papers Series: Volume 60

#### **Decision Tree**

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- 1. Explore *Risk Simulator's* Decision Tree module
- 2. Learn about running Monte Carlo risk simulation on a decision tree
- 3. Find out what the Bayesian analysis tool is used for
- 4. Learn how to compute the Expected Value of Perfect Information (EVPI), MINIMAX and MAXIMIN Analysis, as well as the Risk Profile and the Value of Imperfect Information
- 5. See how sensitivity charts and scenario tables can be useful
- 6. Learn how to generate utility functions

*"What can you do with decision tree models?"* 

### **Contact Us**

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admin@realoptionsvaluation.com www.realoptionsvaluation.com www.rovusa.com *Risk Simulator* | *Decision Tree* runs the *Decision Tree* module (Figure 1), which is used to create and value decision tree models. Additional advanced methodologies and analytics are also included:

- Decision Tree Models
- Monte Carlo risk simulation
- Sensitivity Analysis
- Scenario Analysis
- Bayesian (Joint and Posterior Probability Updating)
- Expected Value of Information
- MINIMAX
- MAXIMIN
- Risk Profiles



Figure 1. ROV Decision Tree (Decision Tree)

The following are some main quick getting started tips and procedures for using this intuitive tool:

- There are 11 localized languages available in this module and the current language can be changed through the *Language* menu.
- *Insert Option* nodes or *Insert Terminal* nodes by first selecting any existing node and then clicking on the option node icon (square) or terminal node icon (triangle), or use the functions in the *Insert* menu.
- Modify individual Option Node or Terminal Node properties by double-clicking on a node. Sometimes when you click on a node, all subsequent child nodes are also selected (this allows you to move the entire tree starting from that selected node). If you wish to select only that node, you can have to click on the empty background and click back on that node to select it individually. Also, you can move individual nodes or the entire tree started from the selected node depending on the current setting (right-click, or in the Edit menu, select More Nodes Individually or More Nodes Together).

- The following are some brief descriptions of the things that can be customized and configured in the node properties user interface. It is simplest to try different settings for each of the following to see its effects in the Strategy Tree:
  - *Name*: Name shown above the node.
  - *Value*: Value shown below the node.
  - o Excel Link: Links the value from an Excel spreadsheet's cell.
  - o *Notes*: Notes can be inserted above or below a node.
  - o Show in Model: Show any combinations of Name, Value, and Notes.
  - o Local Color versus Global Color. Node colors can be changed locally to a node or globally.
  - Label Inside Shape: Text can be placed inside the node (you may need to make the node wider to accommodate longer text).
  - o Branch Event Name. Text can be placed on the branch leading to the node to indicate the event leading to this node.
  - *Select Real Options*: A specific real option type can be assigned to the current node. Assigning real options to nodes allows the tool to generate a list of required input variables.
- *Global Elements* are all customizable, including elements of the Strategy Tree's *Background, Connection Lines, Option Nodes, Terminal Nodes,* and *Text Boxes.* For instance, the following settings can be changed for each of the elements:
  - o Font settings on Name, Value, Notes, Label, Event names.
  - o Node Size (minimum and maximum height and width).
  - o Borders (line styles, width, and color).
  - o *Shadow* (colors and whether to apply a shadow or not).
  - o Global Color.
  - o Global Shape.
- The *Edit* menu's *View Data Requirements Window* command opens a docked window on the right of the Strategy Tree such that when an option node or terminal node is selected, the properties of that node will be displayed and can be updated directly. This feature provides an alternative to double-clicking on a node each time.
- Example Files are available in the File menu to help you get started on building Strategy Trees.
- *Protect File* from the *File* menu allows the Strategy Tree to be encrypted with up to a 256-bit password encryption. Be careful when a file is being encrypted because if the password is lost, the file can no longer be opened.
- *Capturing the Screen* or printing the existing model can be done through the *File* menu. The captured screen can then be pasted into other software applications.
- *Add, Duplicate, Rename,* and *Delete a Strategy Tree* can be performed through right-clicking the Strategy Tree tab or the *Edit* menu.
- You can also *Insert File Link* and *Insert Comment* on any option or terminal node, or *Insert Text* or *Insert Picture* anywhere in the background or canvas area.
- You can *Change Existing Styles* or *Manage and Create Custom Styles* of your Strategy Tree (this includes size, shape, color schemes, and font size/color specifications of the entire Strategy Tree).
- Insert Decision, Insert Uncertainty, or Insert Terminal nodes by selecting any existing node and then clicking on the decision node icon (square), uncertainty node icon (circle), or terminal node icon (triangle), or use the functionalities in the Insert menu.
- Modify individual Decision, Uncertainty, or Terminal nodes' properties by double-clicking on a node. The following are some additional unique items in the *Decision Tree* module that can be customized and configured in the node properties user interface:
  - Decision Nodes: *Custom Override* or *Auto Compute* the value on a node. The automatically compute option is set as default and when you click *Run* on a completed Decision Tree model, the decision nodes will be updated with the results.
  - o Uncertainty Nodes: *Event Names, Probabilities,* and *Set Simulation Assumptions.* You can add probability event names, probabilities, and simulation assumptions only after the uncertainty branches are created.
  - Terminal Nodes: *Manual Input, Excel Link*, and *Set Simulation Assumptions*. The terminal event payoffs can be entered manually or linked to an Excel cell (e.g., if you have a large Excel model that computes the payoff, you can link the model to this Excel model's output cell) or set probability distributional assumptions for running simulations.
- *View Node Properties Window* is available from the *Edit* menu and the selected node's properties will update when a node is selected.

- The Decision Tree module also comes with the following advanced analytics:
  - o Monte Carlo Simulation Modeling on Decision Trees
  - o Bayes Analysis for obtaining posterior probabilities
  - Expected Value of Perfect Information, MINIMAX and MAXIMIN Analysis, Risk Profiles, and Value of Imperfect Information
  - o Sensitivity Analysis
  - o Scenario Analysis
  - o Utility Function Analysis

## Simulation Modeling

This tool runs Monte Carlo risk simulation on the decision tree (Figure 2). It allows you to set probability distributions as input assumptions for running simulations. You can either set an assumption for the selected node or set a new assumption and use this new assumption (or use previously created assumptions) in a numerical equation or formula. For example, you can set a new assumption called Normal (e.g., normal distribution with a mean of 100 and standard deviation of 10) and run a simulation in the decision tree, or use this assumption in an equation such as (100\*Normal+15.25).

Create your own model in the numerical expression box. You can use basic computations or add existing variables into your equation by double-clicking on the list of existing variables. New variables can be added to the list as required either as numerical expressions or assumptions.



Figure 2. ROV Decision Tree (Simulation Results)

## **Bayes Analysis**

The Bayesian analysis tool (Figure 3) can be used on any two uncertainty events that are linked along a path. For instance, in the example on the right in Figure 3, uncertainties A and B are linked, where event A occurs first in the timeline and event B occurs second. First Event A is Market Research with 2 outcomes (Favorable or Unfavorable). Second Event B is Market Conditions also with 2 outcomes (Strong and Weak). This tool is used to compute joint, marginal, and Bayesian posterior updated probabilities by entering the prior probabilities and reliability conditional probabilities; or reliability probabilities can be computed when you have posterior updated conditional probabilities. Select the relevant analysis desired and click on *Load Example* to see the sample inputs corresponding to the selected analysis and the results shown in the grid on the right, as well as which results are used as inputs in the decision tree in the figure.

## **Quick Procedures**

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- **STEP 1:** Enter the names for the first and second uncertainty events and choose how many probability events (states of nature or outcomes) each event has.
- **STEP 2:** Enter the names of each probability event or outcome.
- **STEP 3:** Enter the second event's prior probabilities and the conditional probabilities for each event or outcome. The probabilities must sum to 100%.

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TEP 1: Enter the	names for the f	irst and second u	ncertainty events	and choose how	w many probability events	(states of nature or outcome	;)	Prob (Strong)	45.00%
ach event has.								Prob (Weak)	55.00%
rst Event Name		Mark	et Research	Probability	Events or States	2 🌲		Prob (Favorable   Strong)	60.00%
Second Event Name Market Conditions Probability Events or States				2 🛋		Prob (Favorable   Weak)	30.00%		
TTD 2: Take the server of and and all the surger of and any transfer of and any transfer of an				Events or states			Prob (Unfavorable   Strong)	40.00%	
STEP 2: Enter the names of each probability event or outcome.						Prob (Unfavorable   Weak)	70.00%		
States Market Research Market Conditions							Load Example	Joint and Marginal Probabilities	
1		Favorable			Strong			Prob (Favorable)	43.50%
2		Unfavorabl	e		Weak		Compute	Prob (Unfavorable)	56.50%
								Prob (Strong ∩ Favorable)	27.00%
								Prob (Weak ∩ Favorable)	16.50%
						200		Prob (Strong ∩ Unfavorable)	18.00%
EP 3: Enter the obabilities must	second event's sum to 100%.	prior probabilities	and the condition	hal probabilities f	or each event or outcome.	The		Prob (Weak o Unfavorable)	38.50%
				Co	nditional Probabilities (Relia	bilities) Saved Model		Posterior or Updated Probabilities	
Events	Drior D(x)	Favorable	Unfavorable	SUM		Name		Prob (Strong   Favorable)	62.07%
events		CO DOD/	do ooo/	100.000/			ADD	Prob (Weak   Favorable)	37.93%
strong	45.00%	60.00%	40.00%	100.00%				Prob (Strong   Unfavorable)	31.86%
Weak	55.00%	30.00%	70.00%	100.00%				Prob (Weak   Unfavorable)	68.14%
SUM	100.00%						DEL		

Figure 3. ROV Decision Tree (Bayes Analysis)

# Expected Value of Perfect Information, MINIMAX and MAXIMIN Analysis, Risk Profiles, and Value of Imperfect Information

This tool (Figure 4) computes the Expected Value of Perfect Information (EVPI), MINIMAX and MAXIMIN Analysis, as well as the Risk Profile and the Value of Imperfect Information. To get started, enter the number of decision branches or strategies under consideration (e.g., build a large, medium, or small facility), the number of uncertain events or states of nature outcomes (e.g., good market, bad market), and the expected payoffs under each scenario.

The Expected Value of Perfect Information (EVPI), assuming you had perfect foresight and knew exactly what to do (through market research or other means to better discern the probabilistic outcomes), computes if there is added value in such information (i.e., if market research will add value) as compared to more naive estimates of the probabilistic states of nature. To get started, enter the number of decision branches or strategies under consideration (e.g., build a large, medium, or small facility) and the number of uncertain events or states of nature outcomes (e.g., good market, bad market), and enter the expected payoffs under each scenario.

MINIMAX (minimizing the maximum regret) and MAXIMIN (maximizing the minimum payoff) are two alternate approaches to finding the optimal decision path. These two approaches are not used often but still provide added insight into the decision-making process. Enter the number of decision branches or paths that exist (e.g., building a large, medium, or small facility), as well as the uncertainty events or states of nature under each path (e.g., good economy vs. bad economy). Then, complete the payoff table for the various scenarios and Compute the MINIMAX and MAXIMIN results. You can also click on *Load Example* to see a sample calculation.



Figure 4. ROV Decision Tree (EVPI, MINIMAX, Risk Profile)

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## Sensitivity

Sensitivity analysis (Figure 5) on the input probabilities is performed to determine the impact of inputs on the values of decision paths. First, select one Decision Node to analyze, and then select one probability event to test from the list. If there are multiple uncertainty events with identical probabilities, they can be analyzed either independently or concurrently.

The sensitivity charts show the values of the decision paths under varying probability levels. The numerical values are shown in the results table. The location of crossover lines, if any, represents at what probabilistic events a certain decision path becomes dominant over another.

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Sensitivity analysis on the input probabilities is performed to determine its impact on the values of decision paths. First, select one Decision Node to analyze below, then select one probability event to test from the list. If there are multiple uncertainty events with identical probabilities, they can be analyzed either independently or concurrently.											
Step 1: Select one or more Decision paths to Uncertainty Nodes and Terminal Nodes Step 4: Enter the input sensitivity range											
analyze from the list.	Node and ID	Probability	*	Uncertainty Probabilities	FROM	0.00%	TO 100.00%	STEP SIZE	5.00%		
nature) or ONE Terminal node's payoff	Critical? [1.1]			Terminal Payoffs	FROM		то	STEP SIZE			
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change all identical probabilities/payoffs concurrently.	Critical? [1.2]			Probabilities	0.00%	5.00%	10.00%	15.00%	20.00%		
<ul> <li>Analyze probabilities/payoffs in groups</li> </ul>	Completion Time C	30.00%		Critical?	122.20	124.18	126.16	128.14	130.12		
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Figure 5. ROV Decision Tree (Sensitivity Analysis)

## Scenario Tables

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Scenario tables (Figure 6) can be generated to determine the output values given some changes to the input. You can choose one or more Decision paths to analyze (the results of each path chosen will be represented as a separate table and chart) and one or two Uncertainty or Terminal nodes as input variables to the scenario table.

## **Quick Procedures**

- Select one or more Decision paths to analyze from the list below.
- Select one or two Uncertainty Events or Terminal Payoffs to model.
- Decide if you wish to change the event's probability on its own or all identical probability events at once.
- Enter the input scenario range.



Figure 6. ROV Decision Tree (Scenario Tables)

# **Utility Function Generation**

Utility functions (Figure 7), or U(x), are sometimes used in place of expected values of terminal payoffs in a decision tree. They can be developed two ways: using tedious and detailed experimentation of every possible outcome or an exponential extrapolation method (used here). They can be modeled for a decision maker who is risk-averse (downsides are more disastrous or painful than an equal upside potential), risk-neutral (upsides and downsides have equal attractiveness), or risk-loving (upside potential is more attractive). Enter the minimum and maximum expected value of your terminal payoffs and the number of data points in between to compute the utility curve and table.

If you had a 50:50 gamble where you either earn X or lose X/2 versus not playing and getting a 0 payoff, what would this X be? For example, if you are indifferent between a bet where you can win 100 or lose 50 with equal probability compared to not playing at all, then your X is 100. Enter the X in the Positive Earnings box. Note that the larger X is, the less risk-averse you are, whereas a smaller X indicates that you are more risk-averse.

Enter the required inputs, select the U(x) type, and click *Compute Utility* to obtain the results. You can also apply the computed U(x) values to the decision tree to rerun it, or revert the tree back to using expected values of the payoffs.

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Utility functions, or U(x), are sometimes used in place of expected values of terminal payoffs in a decision tree. U(x) can be developed two ways: using tedious and detailed experimentation of every possible outcome or an exponential extrapolation method (used here). They can be modeled for a decision maker who is risk-averse (downsides are more disastrous or painful than an equal upside potential), risk-neutral (upsides and downsides have equal attractiveness), or risk-loving (upside potential is more attractive). Enter the minimum and maximum expected value of your terminal payoffs and the number of data points in between to compute the utility curve and table.												
Minimum Expected Value	34.00	Min expected value	ue payoff to generat	e the start of the U(x) curve	EV	U1(x)	-	Terminal Payoffs	U1(x) 🔺			
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					42.1939	0.1762		286.0000	0.7311			
U(x) Points to Compute	50	Number of steps	to take between the	min and max U(x)	50.3878	0.2066		362.0000	0.8103			
If you had a 50:50 gamble where	vou either e	arn \$X or lose -\$X/2	versus not plaving a	nd getting a \$0 payoff, what would	58.5816	0.2359		44.0000	0.1830			
this \$X be? For example, if you are	e indifferent	between a bet whe	re you can win \$100	or lose -\$50 with equal probability	66.7755	0.2641		48.0000	0.1978 =			
compared to not playing at all, the	en your X is s	\$100. Enter the X in	the Positive Earning	s box below. Note that the larger X is,	74.9694	0.2913		56.0000	0.2268			
the less risk-averse you are, when	eas a smalle	er x indicates that yo	ou are more risk-ave	se.	83.1633	0.3175		306.0000	0.7547			
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Positive Earnings	217.75	obtain the results	. You can also apply	the computed U(x) values to the	99.5510	0.3669		435.5000	0.8647			
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U3: Risk-Averse Utility Function (Calibrated between 0 and 1) U1(x) U4(x), U6(x) & U9(x)												
U4: Risk-Averse Utility Function ( U5: Risk-Neutral Utility Function (	etween 0 and 100)		entral entral a cotral									
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Figure 7. ROV Decision Tree (Utility Functions)